Understanding the Behavior of Complex Structures Under Load

PART TWO

By Joop van der Lelij

Today’s design engineers employ powerful methods to understand and predict the behavior of complex structures under defined loads. But even such well-known theoretical methods as Finite Element Analysis, the mathematical computer program to calculate deformations and stresses anywhere in a structural model, are really only starting points. Values obtained for deflections and stresses must still be compared to allowable values. In this second half of a two-part article, Joop van der Lelij, a computer specialist in the DuPont Engineering Polymers Computer-Aided Technical Service [CATS] group in Geneva, describes rules to derive allowable stresses.

Static Load

The allowable stress is the smallest value of:

\[
\sigma_{\text{all}} = S \times \sigma_{\text{ult}}
\]

Where: 
- \( S \) = safety factor (see Table 1)
- \( \sigma_{\text{ult}} \) = ultimate tensile strength; first maximum of a stress-strain curve, measured at design conditions (temperature, exposure time, moisture conditions as for actual part, see Figure 4). For very long time periods, creep strength should also be considered.
- \( E \) = modulus of elasticity at design conditions (slope of stress strain curve at 0 stress)
- \( \varepsilon_{\text{brk}} \) = elongation at break design conditions (%/100)
- \( \text{SCF} \) = stress concentration factor (minimum value is 4)

Stress Concentration Factor

If a structure shows discontinuities in geometry, which is almost always the case for end-use parts, the stresses at these discontinuities are higher than the stresses in the adjacent structural parts.

The ratio of the highest discontinuity stress (peak stress) and the mean stress which could theoretically be expected is defined as the stress concentration factor (SCF). For a flat plate with a small circular hole in it, the SCF already has a value of 3 (see Figure 5).

To avoid peak stresses, which can cause local strains higher than elongation at break for brittle materials (resulting in rupture), the second requirement for the allowable stress is added, including a minimum SCF of 4.
This factor of 4 is chosen because not all fillets in a part can be assumed to be as good as a circular hole. For poor fillets, the SCF value is estimated at close to 6.

The highest discontinuity stresses are probably never located in the same areas of a part where strength has been diminished as a result of processing. S and SCF are not both applicable at the same time then, or at the same location. Figure 6 shows the effects of a limitation to the maximum strain.

**Dynamic Load**

Resistance of components to dynamic loads and to static loads differ, as do the types of failure caused by the two types of loading. At highly stressed points, typically a void, a crack will start and grow. Fatigue strength of parts, including ribs where applicable, should be measured for alternating and semi alternating loads under different conditions.

Use FEA to calculate the stresses for the maximum load on a part; then check the part on dynamic test apparatus to show the allowable number of cycles for a given load. By varying the load, a curve can be obtained showing strength as a function of the number of cycles. Figure 7 shows an example of such curves. Ideally, the calculations should be carried out using the linear-elastic approach.

**Impact Load**

Because thermoplastics have properties—tensile strength, modulus of elasticity, elongation at break—that vary considerably with rate of deformation (strain rate), methods of assessing their resistance to impact loading have never been developed completely. Moreover, deformation speed is not the same throughout a structure. Further assessment of impact testing, therefore, is needed.

**Allowable Stress in DuPont Polymers**

The above rules and guidelines for allowable stresses are valid for plastics materials with a modulus of elasticity of more than 1000 MPa. This means that they are applicable to DuPont Delrin® acetal homopolymer, Zytel® nylon, Zytel® glass reinforced nylon (GRZ), Minlon® mineral-reinforced nylon, and Rynite® glass-reinforced PBT and PET polyester resins. It is also assumed that the equivalent stresses are computed using handbook formulae, or by the finite element method using linear elastic theory.
In calculations using isotropic material behavior for glass-reinforced materials, the modulus of elasticity should be set to 85 percent of the test-bar value to correct for non optimally oriented fibers.

*Behavior of Hytrel*®

Hytrel® thermoplastic polyester elastomer is relatively soft and behaves more like an elastomeric material. For Hytrel®, Poisson’s ratio has a value close to 0.5, which causes numerical problems for the standard elements used in FEA. To overcome this difficulty, special hyper-elastic elements for use in FEA are now available. These elements use the so-called “Mooney-Rivlin” coefficients, which are derived from the stress-strain curve.

It appears that the lower the modulus of elasticity of a material, the more difficult it is to reproduce its physical properties in subsequent production batches. This influences the accuracy of calculation results. However, it is possible to achieve results for Hytrel® which are accurate to within 10 to 20 percent.

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This article was originally published in the 1991 issue of “Engineering Design” magazine.

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